

# The Thirty-Second Annual Eastern Shore High School Mathematics Competition

November 5, 2015

## Individual Contest Exam

### Instructions

There are twenty problems on this exam. Select the best answer for each problem.

Your score will be the number of *correct* answers that you select.

**There is no penalty for incorrect answers.**

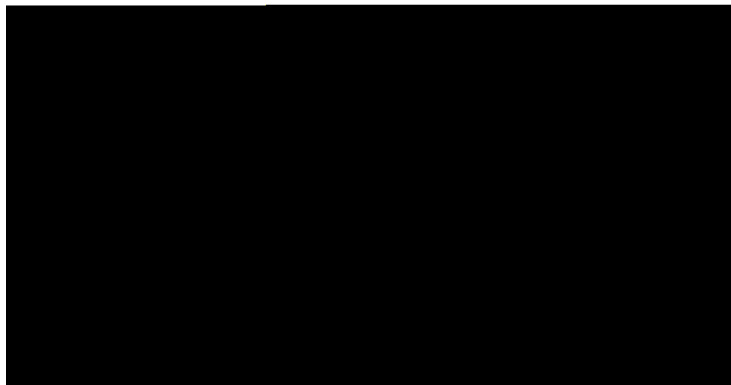
The use of a calculator is **not** permitted on this exam.

In the event of tie scores, #18, #19 and #20 will be used as tiebreakers.

1. A group of zombies moved into the zombie-free town of Sittingducks on February 1st with a master plan to turn a certain number of the townspeople into zombies each day. On February 2nd, the zombies turned 1 of the townspeople into a zombie; the next day, they turned 4 of the townspeople into zombies, then 7 into zombies the day after that, and so on. (In other words, each day they turned 3 more of the townspeople into zombies than they did the day before.) At the end of the day on February 21st, there were 600 total zombies in the town. How many zombies moved into Sittingducks on February 1st?

- a. 7      b. 8      c. 9      d. 10      e. 11

2. Brandon lives in a city where the streets are laid out on a grid pattern; the streets are either perpendicular or parallel to each other. Additionally, the distance between any two parallel streets is an integer number of miles. If Brandon could travel along a straight line from his school to his house, he would only need to cover a distance of 13 miles to get home from school, but Brandon can only travel on streets. What is the minimum number of miles Brandon has to travel in order to get home from school?



- a. 13      b. 15      c. 17      d. 19      e. 21

3. A cube is inscribed in a sphere of radius 1. If the surface area of the cube is  $A$  square units, which of the following is true?

- a.  $7 < A < 8$       b.  $8 < A < 9$       c.  $9 < A < 10$       d.  $10 < A < 11$       e.  $11 < A < 12$

4. Suppose the  $xy$ -plane is reflected through the line  $y = x$ , then rotated  $45^\circ$  clockwise about the origin. What is the image of the point  $(0; 2)$  under this pair of transformations?

- a.  $(\sqrt{2}; \sqrt{2})$       b.  $(-\sqrt{2}; \sqrt{2})$       c.  $(\sqrt{2}; -\sqrt{2})$       d.  $(2; 2)$       e.  $(-2; -2)$

5. Suppose  $\sin \theta = \frac{2}{3}$  and  $\cos \theta > 0$ . Then  $\frac{1}{\tan \theta}$  equals

- a.  $\frac{\sqrt{13}}{2}$       b.  $\frac{\sqrt{5}}{2}$       c. 0      d.  $\frac{\sqrt{5}}{2}$       e.  $\frac{\sqrt{13}}{2}$

6. Bowl I has 15 marbles: 5 green, 6 yellow, and 4 red. Bowl II has 12 marbles: 4 green, 4 yellow, and 4 red. A marble is selected from each bowl. Find the probability that both marbles are yellow.

- a.  $\frac{2}{25}$       b.  $\frac{2}{15}$       c.  $\frac{10}{27}$       d.  $\frac{1}{2}$       e.  $\frac{11}{15}$

7.  $2015^{2015^{2015}}$  is equivalent to which of the following expressions?

a.  $2015(2015^{2015})$

b.  $5^{2015}13^{2015}31^{2015}$

c.  $(2015 \ 2015)^{2015}$

d.  $\prod_{i=0}^{2015} 2015$

e.  $2015^{5^{2015}13^{2015}31^{2015}}$

8. The repeating decimal  $2.\overline{015}$  can be expressed as the quotient of which two integers?

a. 2000 and 983

b. 2015 and 1000

c. 2011 and 999

d. 2013 and 999

e. 2015 and 999

9. Which of the following functions is its own inverse function?

a.  $f(x) = \frac{2x - 7}{5x - 2}$

b.  $f(x) = \frac{3x + 5}{3x - 5}$

c.  $f(x) = \frac{x - 5}{2x - 8}$

d.  $f(x) = \frac{9x + 2}{7x - 4}$

e.  $f(x) = \frac{4x - 5}{5x - 5}$

10. Given  $A \cap B = A$ , which of the following statements must be true:

a.  $A \setminus B = A$

b.  $A \subset B$

c.  $B \subset A$

d.  $A = B$

e. None of the above.

11. Jimmy knows the following about his three friends. First, he knows that if Jerry is not the tallest of the three, then Esther is. Second, he knows that if Esther is not the shortest, then Rick is the tallest. List the friends from tallest to shortest:

a. Esther, Jerry, Rick

b. Esther, Rick, Jerry

c. Rick, Esther, Jerry

d. Jerry, Rick, Esther

e. Can't be determined

12. What is the units digit of  $2015^{2016} - 2016^{2015}$ ?

a. 1

b. 3

c. 5

d. 7

e. 9

13. The shortest side of a right triangle has length 2, and its hypotenuse has length 3. What is the measure of the triangle's smallest angle?

- a. 30
- b.  $\arctan \frac{2}{0.8}$
- c.  $\arctan \frac{2}{1.25}$
- d.  $\arcsin(1.5)$
- e.  $\arccos(1.5)$

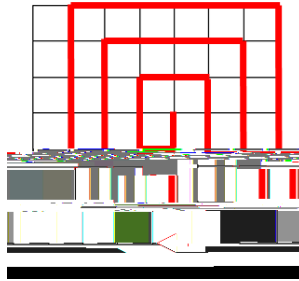
14. The sum of all integer-valued solutions to the equation  $\frac{2}{x-13} > \frac{8}{9}$  is

- a. 48
- b. 52
- c. 65
- d. 77
- e. 90

15. A sample consists of six test scores: 86, 72, 92, 90, 78, and 74. The highest possible score on the test was 100. Suppose that one more test score is selected. Find the value of the seventh test score so that the new sample mean is equal to the new median.

- a. 78
- b. 82
- c. 91
- d. Both a and c
- e. None of these

19. David's morning walk follows a precise pattern, a right-angled spiral, as shown in the figure below. The figure is drawn to scale but is not complete. If David walks 5 feet before his first turn, how many turns has he made when he has walked 300 feet?



- a. 10 turns      b. 12 turns      c. 13 turns      d. 14 turns      e. 15 turns
20. At the  $n$ th minute, a square of side length  $n$  cm is placed on the screen so that it does not overlap any of the previously placed squares. After 10 minutes, how much area of a 30 cm  $\times$  45 cm screen remains uncovered?
- a. 385 square cm      b. 965 square cm      c. 1000 square cm      d. 1295 square cm      e. 1385 square cm