

# The Thirty-Fourth Annual Eastern Shore High School Mathematics Competition

November 9, 2017

## Individual Contest Exam

### Instructions

There are twenty problems on this exam. Select the best answer for each problem.

Your score will be the number of *correct* answers that you select.

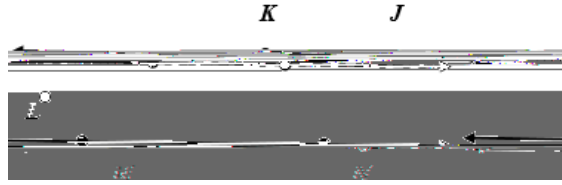
**There is no penalty for incorrect answers.**

The use of a calculator is **not** permitted on this exam.

No computational work is required for any of your multiple choice responses.

However, in the event of tie scores, after the multiple choice responses have been checked for problems 1-20, the responses and/or written computational work on the enclosed form for problems #18, #19 and #20 will then be used as tiebreakers.

1. In the figure below point L is between  $J'K$  and  $M'N$  and  $J'K \parallel M'N$ . If  $s = m \angle JKL + m \angle KLM + m \angle LMN$ , then



- a.  $s < 360^\circ$       b.  $s > 360^\circ$       c.  $s = 360^\circ$       d.  $360^\circ < s < 540^\circ$       e. cannot be determined
2. A central angle,  $\theta$ , in a circle of radius 10 inches intercepts an arc of length 40 inches. What is the radian measure of  $\theta$ ?
- a. 0.25 radians  
 b. 2 radians  
 c. 3.5 radians  
 d. 4 radians  
 e. cannot be determined from the given information
3. If  $\ln(x+1) + 2\ln(x+2) - \ln(x^2 + 5x + 4) = \ln \frac{2}{3}$ , then
- a.  $x = \frac{1}{2}$       b.  $x = \sqrt{2}$       c.  $x = 2$       d.  $x = 2\sqrt{2}$       e.  $x = 4$
4. Let  $f(x) = ax + b$  and  $h(x) = x^2$ . Find all ordered pairs  $(a; b)$  so that  $f(h(x)) = h(f(x))$  for all  $x$ .
- a. (0,0)  
 b. (1,0) and (0,1)  
 c. (1,0), (0,1), and (1,1)  
 d. (0,0), (1,0), and (0,1)  
 e. (0,0), (1,0), (0,1), and (1,1)

5.  $\tan^{-1}(\sin^{-1}(x))$  is equal to

- a.  $\frac{x}{\sqrt{1-x^2}}$   
 b.  $\frac{x}{1-x^2}$   
 c.  $x$   
 d.  $\frac{x}{1-x^2}$   
 e.  $\frac{x}{\sqrt{1-x^2}}$

6. Which of the following represents the set of elements in X or Y or Z but not all three simultaneously?  
(Note that  $A^c$  denotes the complement of the set A.)

I:  $(X \cup Y \cup Z) \setminus (X \cap Y \cap Z)^c$

II:  $(X \setminus (Y \cap Z)^c) \cup (Y \setminus (X \cap Z)^c) \cup (Z \setminus (Y \cap X)^c)$

III:  $(X \setminus (Y^c \cap Z^c)) \cup (Y \setminus (X^c \cap Z^c)) \cup (Z \setminus (Y^c \cap X^c))$

IV:  $(X \setminus (Y^c \cap Z^c)) \cup (Y \setminus (X^c \cap Z^c)) \cup (Z \setminus (Y \cap X)^c)$

V:  $(X \setminus (Y \cap Z)^c) \cup (Y \setminus (X \cap Z)^c) \cup (Z \setminus (Y \cap X)^c)$

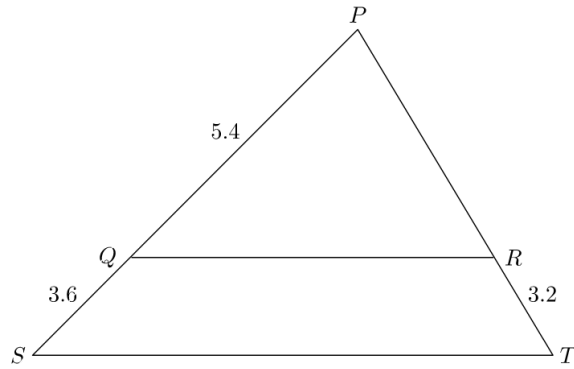
- a. I only  
 b. I, IV and V  
 c. I, II and III  
 d. I, II and IV  
 e. none of the above
7. A sample consists of four test scores: 85, 72, 91, and 78. Suppose that one more test score is selected. For which value of the 5th test score is the new sample mean equal to the new median?

- a. 64      b. 78      c. 81      d. 99      e. both a and d

8. A group of students at an event is made up of 28 females and 22 males. Twelve of the males are math majors and ten are computer science majors. Twenty of the females are computer science majors and the remaining are math majors. If one of these students is selected at random, what is the probability that the student is a math major if it is known that the student is female?

- a.  $\frac{8}{50}$       b.  $\frac{8}{28}$       c.  $\frac{20}{50}$       d.  $1 - \frac{12}{22}$       e.  $\frac{1}{2}$

11. In the image shown below,  $QR$  is parallel to  $ST$ . What is the length of  $PR$ ?



- a. 2:13      b. 4:8      c. 5      d. 6.075      e. 6:5

12. Which one of the following has the value of 2017?

- a.  $2018^{\log_2(2017 \cdot 2^{11} + 2^4 + 2^3 + 2^2 + 2^1 + 2^0 + 2^0)}$   
 b.  $2017^{\log_2(2018 \cdot 2^{11} + 2^4 + 2^3 + 2^2 + 2^1 + 2^0)}$   
 c.  $2017^{\log_2(2017 \cdot 2^{11} + 2^4 + 2^3 + 2^2 + 2^1 + 2^1 + 2^0)}$   
 d.  $(\log_2(2017 \cdot 2^{11} + 2^4 + 2^3 + 2^2 + 2^1 + 2^1 + 2^0))^{\log_{2017}(2017^{2018})}$   
 e.  $(\log_2(2017 \cdot 2^{11} + 2^4 + 2^3 + 2^2 + 2^1 + 2^1 + 2^0))^{\log_{2018}(2018^{2017})}$

13. 41

16. An equilateral triangle is inscribed in a circle of radius  $r$ . Between each of the sides and the circumference a circle is drawn tangent to each. What is the sum of the areas of these three circles?



- a.  $\frac{3}{4} r^2$     b.  $\frac{3}{16} r^2$     c.  $\frac{9}{4} r^2$     d.  $\frac{9}{16} r^2$     e. none of these
17. Suppose it is 11:00 AM right now. What time will it be 2017 hours from now?
- a. 4:00 AM    b. 7:00 AM    c. 12:00 Noon    d. 6:00 PM    e. none of these
18. Ruby doesn't like grapes, so while no one was looking, she tried to give her grapes to a few of her princess dolls. She always likes to be fair, so she made sure to share the grapes equally among her dolls. However ...
- When she tried to share them equally among 3 dolls, there was 1 grape left over. So, she brought another doll to the table ...
- When she tried to share them equally among 4 dolls, there was still 1 grape left over. So, she went and found yet *another* doll
- When she tried to share them equally among 5 dolls, there was, once again, 1 grape left over.
- At this point, she gave up, dejectedly ate one grape herself (she doesn't like to be wasteful), and then shared the rest equally among the 5 dolls.
- Assuming she started with fewer than 100 grapes, how many grapes did each of Ruby's five princess dolls end up with?
- a. 6    b. 8    c. 9    d. 10    e. 12
19. Martian potatoes are 99% water. Suppose we have 50 pounds of Martian potatoes. If the potatoes are dried so that they are now only 98% water, the new total weight of the potatoes falls in what range?
- a. 20 lbs. to 23 lbs.  
 b. 24 lbs. to 27 lbs.  
 c. 28 lbs. to 31 lbs.  
 d. 32 lbs. to 35 lbs.  
 e. none of these
20. Let  $1, 3, 5, \dots$  and  $9, 13, 17, \dots$  be two arithmetic sequences, and consider just the first 2017 terms of each sequence. How many numbers do the two sequences have in common?
- a. 109    b. 351    c. 621    d. 1007    e. 1344